DO FOREIGN EXCHANGE RETURN REGRESSIONS CONVEY USEFUL INFORMATION ON RETURN PREDICTABILITY?*

SEONGMAN MOON
Chonbuk National University

CARLOS VELASCO
Universidad Carlos III de Madrid

This paper shows the possibility that the estimates from foreign exchange return regressions contain huge noise which makes it difficult to extract useful information about the predictability of foreign excess returns, in particular, if exchange rates are generated from a typical present value model with a near unity discount factor. The main reason is that the present value model induces a large bias in the estimation of the regressions accompanied by a high variability of the estimates. We also confirm that the volatility and persistence of both the spot return and the forward premium generated from the present value model are consistent with data.

Key words: present value model, discount factor, contemporaneous correlation, forward premium puzzle.

JEL classification: F31, C13.

Uncovered interest parity (UIP), one of major building blocks in international macroeconomic models, states that the expected excess return on foreign currency must be equal to zero under the assumptions of rational expectations and risk neutral preferences. Numerous studies, however, have persistently found negative estimates from the regression of the change in the logarithms of the spot exchange rate (the spot return) on the lagged forward premium, which are much far away from the value of one under UIP. These results imply that a higher interest rate currency tends to appreciate, rather than to depreciate. Two popular economic explanations have been considered to explain these puzzling negative values: one is the rational expectations risk premium and the other is expectational

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errors[1]. Many researchers have built economic models which incorporate each of the two explanations since these two measures are not observable. Therefore, they need certain criteria to judge the performance of their models. One example is Fama (1984)'s volatility relations.

Fama (1984) illustrated the magnitude of variability in the expected excess return using the return regression test under the assumption of rational expectations. One relationship he derived is that if the slope coefficient is less than one half in the return regression, the variance of the rational expectations risk premium (the expected excess return) should be greater than that of the expected change in exchange rates (the expected spot return). Since then, most studies in the risk premium literature have used this condition to judge the performance of international asset pricing models for explaining the observed behavior of forward and spot rates. However, almost all models miserably fail in generating a high volatility of the risk premium, which refers to the forward premium puzzle[2]. The implicit presumption in the literature was that the estimated slope coefficient in the regression of the spot return on the forward premium would accurately convey information about the predictability of excess returns.

Our paper questions this presumption on the information content of the regression[3]. We analytically show that the present value model of exchange rates with a near unity discount factor generates a large magnitude of the relative variance between the spot return and the forward premium, consistent with the data. We then show that this large relative variance is closely linked to imprecise estimation of the return regression. Our results imply that caution is needed in relating the magnitude of the estimated slope coefficient to the amount of the risk premium (or a deviation from UIP). Nevertheless, we find that this large variation itself does not lead to the overrejection of the conventional *t*-test for the estimated slope coefficient in finite samples, in contrast to small sample bias problems in the regressions of stock return predictability[4]. The main reason is that the contemporaneous correlation between innovations to foreign exchange excess returns and to the forward premium is close to zero in our sample. Finally, we conclude that it should not be infrequent to find *insignificant negative* estimates in practice if the exchange rates are generated from the typical present value models with the near unity discount factor.

(1) See Engel (1996) and Lewis (1995) for surveys of the literature about these puzzling results. For recent contributions, see Verdelhan (2010) and Kim, Moon, and Velasco (2017). On the other hand, several papers concern with small sample biases based on either econometric theories or peso problems. See for example Evans and Lewis (1995), Baillie and Bollerslev (2000), and Maynard and Phillips (2001).

(2) Recently, Alvarez, Atkeson, and Kehoe (2009) and Verdelhan (2010) claim that they are partially successful for relating the cause of the negative estimates to the rational expectations risk premium. On the other hand, Kim, Moon, and Velasco (2017) present empirical evidence that the Volcker monetary policy regime (1979-1987) significantly contributes to building up the forward premium puzzle.

(3) Baillie and Bollerslev (2000) and West (2012) also study a similar issue. In particular, our paper is close to West (2012) whose analysis is also based on the present value model of exchange rates.

(4) Maynard (2006) also reached a similar conclusion, while investigating some statistical problems due to strong persistence of the predictor in the regressions.

1. Spot return regressions in foreign exchange markets

We reproduce the empirical results using the standard test in the literature to motivate our study in this section. Then, we present the present value model of exchange rates to set out explicitly our question in the next section.

Consider the following bivariate regression model,

$$s_t - s_{t-1} = \alpha + \beta(f_{t-1} - s_{t-1}) + u_t, \quad [1]$$

$$f_t - s_t = \delta + \varphi(f_{t-1} - s_{t-1}) + v_t, \quad [2]$$

where s_t is the logarithm of the spot exchange rate, f_t is the logarithm of the forward exchange rate, $s_t - s_{t-1}$ denotes the spot return, $f_{t-1} - s_{t-1}$ is the forward premium, and $\beta = 1$ under UIP. We further assume $0 < \varphi < 1$. The covariance matrix of the error terms, u_t and v_t, is denoted as

$$\Sigma = \begin{pmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{pmatrix}. \quad [3]$$

Although regression [1] has been widely used to test the UIP hypothesis since Fama (1984), it is now well-known that nonzero correlations between u_t and v_t may generate an endogeneity problem. For example, Mankiw and Shapiro (1986) illustrate that the *t*-test based on the conventional asymptotic critical values tends to overreject the null hypothesis if (i) the regressor exhibits strong persistence and (ii) the absolute value of the contemporaneous correlation between disturbances to the dependent variable and to the regressor is high [see also Stambaugh (1999)].

1.1. Standard UIP test

Panel A of Table 1 reports the results from regression [1] using 18 monthly spot and one-month forward exchange rates from the sample period of 1976:1-2011:01[5]. Estimates of β are negative, their difference from the value of one under the null hypothesis is very large, and the conventional *t*-test strongly rejects the UIP hypothesis for most currencies. These results confirm well-known empirical regularities. In contrast, this strong deviation from UIP disappears in the subsample period of 1988-2011. The estimates for most currencies (15 out of the 18 currencies) are not statistically significant as reported in Panel B. These results confirm the empirical findings by Moon and Velasco (2011) and Kim, Moon, and Velasco (2017) and imply that a particular sample period of the 1980s mainly influences the strong predictability of the spot return in the entire sample period[6].

(5) Note that the sample period ends in 1998:12 for the member countries which belong to the European Monetary Union.We use the same data as Burnside (2011) for kindly providing us his data set. See Burnside (2011) for the detailed description of the data set.

(6) Kim, Moon, and Velasco (2017) provide empirical evidence that the Volcker monetary policy regime (1979-1987) significantly influences both conditional and unconditional exchange rate movements: they show that both the delayed overshooting puzzle (related to the conditional behavior of exchange rates) and the forward premium puzzle (related to the unconditional behavior of exchange rates) are only observed in the Volcker monetary policy regime.

A peculiar phenomenon regarding the estimation results for the subsample period is that although the estimated slope coefficients are not statistically significant, the distribution of those estimates is very wide. For example, the largest estimate is 2.31 for the US dollar-Italian Lira (ITL) exchange rate and the smallest estimate is -1.54 for the US dollar-Swiss Franc (CHF) exchange rate. And many estimates (7 out of the 15 currencies) are negative, suggesting a large absolute difference between the estimates and the true value. The main objective of the present paper is to provide a reason why the wide range of these insignificant negative estimates appears in foreign exchange rate data.

Table 1: RESULTS FROM THE CONVENTIONAL t-TEST AND THE CONDITIONAL TEST

Series	$\hat{\beta}$	t-stat	[2.5, 97.5]	R^2	T
Panel A. The entire sample period					
ATS	-1.04	**-2.82**	**[-1.90, 2.00]**	0.01	275
BEF	-0.48	**-1.98**	**[-1.93, 2.03]**	0.00	275
CAD	-0.81	**-2.92**	**[-2.03, 1.92]**	0.00	420
DKK	-0.72	**-3.72**	**[-1.95, 2.01]**	0.01	420
FRF	0.24	-1.29	[-1.94, 2.02]	0.00	275
DEM	-0.70	**-2.35**	**[-1.93, 1.97]**	0.00	275
IEP	0.51	-0.82	[-1.99, 1.96]	0.00	236
ITL	0.25	-1.77	[-2.03, 1.93]	0.00	275
JPY	-2.26	**-4.41**	**[-1.91, 1.97]**	0.02	391
NLG	-1.55	**-3.56**	**[-1.91, 2.01]**	0.02	275
NOK	-0.41	**-2.94**	**[-1.96, 2.00]**	0.00	420
PTE	0.53	**-2.31**	**[-1.91, 2.02]**	0.02	275
ESP	0.91	-0.30	[-1.94, 2.02]	0.03	275
SEK	0.95	-0.10	[-1.97, 1.99]	0.01	420
CHF	-1.37	**-3.87**	**[-1.92, 1.97]**	0.01	420
GBP	-1.55	**-3.94**	**[-1.92, 2.02]**	0.01	420
AUD	-0.93	**-2.45**	**[-1.92, 2.01]**	0.00	313
NZD	-0.99	**-2.31**	**[-2.02, 1.92]**	0.01	313
Panel B. The subsample period					
ATS	-0.53	-1.35	[-1.77, 2.27]	0.00	131
BEF	-0.27	-1.19	[-1.91, 2.01]	0.00	131
CAD	-0.64	-1.85	[-2.05, 1.80]	0.00	276
DKK	-0.32	-1.86	[-1.93, 2.03]	0.00	276
FRF	0.76	-0.24	[-1.79, 2.10]	0.00	131
DEM	-0.41	-1.34	[-2.21, 1.98]	0.00	131

IEP	1.36	0.54	[-1.99, 1.99]	0.03	131
ITL	2.31	1.22	[-1.79, 2.12]	0.03	131
JPY	-1.91	**-2.91**	**[-1.92, 1.96]**	0.01	276
NLG	-0.62	-1.49	[-1.88, 2.24]	0.00	131
NOK	0.23	-1.03	[-1.95, 2.02]	0.00	276
PTE	0.45	-0.83	[-1.94, 1.96]	0.00	131
ESP	2.04	1.22	[-1.82, 2.05]	0.04	131
SEK	1.25	0.41	[-1.94, 2.02]	0.01	276
CHF	-1.54	**-2.58**	**[-1.97, 1.90]**	0.01	276
GBP	-0.13	-1.15	[-1.95, 1.95]	0.00	276
AUD	-1.12	**-2.12**	**[-1.98, 1.94]**	0.00	276
NZD	0.52	-0.41	[-1.92, 2.00]	0.00	276

Note: β is estimated from the regression [1] using monthly foreign excess returns and forward premiums. "*t*-stat" represents the t-statistic of the estimated slope coefficient $\hat{\beta}$. The *t*-statistic in bold means that the null hypothesis, $\beta = 1$, is rejected at the 5% level. We conduct the conditional test by Jansson and Moreira (2006) based on the bivariate regressions [1]-[2]. The fourth column reports the 2.5 and 97.5% quantiles of the distribution obtained using the conditional test. Polk, Thompson, and Vuolteenaho (2006) develop the algorithms for the conditional test which are available on Polk's homepage (*http://personal.lse.ac.uk/POLK/research/work.htm*). The interval of the quantiles in bold means that the null hypothesis is rejected at the 5% level.

Data are provided by Craig Burnside (see Burnside (2011) for the detail). Our sample includes spot and one-month U.S. dollar (USD) prices of the Austrian shilling (ATS), the Belgian franc (BEF), the Canadian dollar (CAD), the Danish Krona (DKK), the French franc (FRF), the Deutsche mark (DEM), the Irish pound (IEP), the Italian lira (ITL), the Japanese yen (JPY), the Dutch guilder (NLG), the Norwegian Krone (NOK), the Portuguese escudo (PTE), the Spanish peseta (ESP), the Swedish krona (SEK), the Swiss franc (SEK), the Swiss franc (CHF), the British pound (GBP), the Australian dollar (AUD), and the New Zealand dollar (NZD). We use monthly observations from 1976:1 to 2011:1.

1.2. A robust test for UIP

The bilateral system of equations [1] and [2] indicates that regression [1] faces an endogeneity problem which may cause difficulty in statistical inference, as long as the contemporaneous covariance σ_{uv} is not zero. To check the robustness of the results in Table 1, we employ Jansson and Moreira (2006)'s conditional test that is robust to the endogeneity problem and most powerful within the class of unbiased tests[7]. As reported in the third and fourth columns in Panel A and B of Table 1, the results from the conditional test are consistent with those from the conventional *t*-test in both samples, suggesting that the endogeneity problem is not severe in the foreign exchange return regressions.

(7) Maynard (2006) first used this test to study if the strong predictability in foreign exchange rate markets is due mainly to some statistical phenomenon and found that both conventional-tests and conditional tests reach the same conclusion.

To further look into it, we estimate the contemporaneous correlations between the innovations to foreign excess returns and to the forward premium using the bivariate regressions [1]-[2] and find that they are close to zero as reported in Table 2: the absolute values of the estimated correlations are less than 0.2 for almost all currencies. This suggests that the conventional inference works well in foreign exchange rate data even if the forward premium is very persistent. Nevertheless, the large variation of the estimated slope coefficient is problematic in that it may not be informative about the true value of the slope coefficient and deserves further investigation. Note that this phenomenon in foreign exchange rate data is distinct from the stock return data: for example, many studies found strong contemporaneous correlations in innovations between the stock return and the dividend price ratio in the bivariate regression of stock return and the dividend price ratio since Nelson and Kim (1993) [see also Campbell and Yogo (2006)].

2. Spot return regressions in a present value model of exchange rates

In this section, we derive the covariance matrix of innovations, Σ, from the typical present value model of exchange rates in order to study the information content in the spot return regression. In the present value model, the logarithm of the spot exchange rate s_t is expressed as a discounted sum of current and expected future fundamentals,

$$s_t = (1-b)\sum_{i=0}^{\infty} b^i E_t[w_{t+i}], \quad [4]$$

where $E_t(\cdot)$ is the mathematical expectation conditional on a time t information set, $0 < b < 1$ represents the discount factor, and w_t represents the linear combination of logarithms of fundamental variables such as money and output [see, Engel and West (2005) for a more general framework]. This relation between the exchange rate and the fundamentals can be obtained from the typical monetary model of exchange rates under the assumptions of no bubbles and UIP[8].

We assume that the fundamental process evolves in the following way[9]

$$\begin{aligned} w_t &= w_{1,t} + w_{2,t}, \\ \Delta w_{1,t} &= \phi \Delta w_{1,t-1} + \eta_{1,t}, \\ w_{2,t} &= w_{2,t-1} + \eta_{2,t}, \end{aligned} \quad [5]$$

(8) In the ad hoc monetary models, a home money market relation is given by

$$m_t - p_t = y_t - \kappa i_t,$$

where $m_t - p_t$ is the logarithm of home real money demand, y_t is the logarithm of home output, i_t is home nominal interest rate, and κ is the interest elasticity of money demand. Foreign money demand can be defined analogously. From home and foreign money market relations, UIP, and covered interest parity, we can derive the present value relation [4] where $w_t = (m_t - y_t) - (m_t^* - y_t^*)$ if purchasing power parity holds and $w_t = (m_t - y_t) - (m_t^* - y_t^*) + s_t + p_t^* - p_t$ if it does not hold.

(9) Our results are robust to other specifications of fundamental processes as long as the key assumption (a near unity discount factor) is maintained. For example, see Section 5 for an alternative process for w_t where the fundamental process is the sum of random walk and stationary components.

where $0 < \phi < 1$ and both $\eta_{1,t}$ and $\eta_{2,t}$ are *iid* with zero mean normal distributions with variance σ_1^2 and σ_2^2, respectively[10]. From equations [4]-[5], both the forecasting error and the forward premium are derived as

$$u_t = s_t - E_{t-1}[s_t] = \frac{1}{1-b\phi}\left(\eta_{1,t} + (1-b\phi)\eta_{2,t}\right),$$
$$f_t - s_t = \frac{(1-b)}{1-b\phi}\phi\Delta w_{1,t}, \quad [6]$$

where the persistence of the forward premium is governed by the parameter ϕ in $w_{1,t}$. Since $\varphi = \phi$, the error term v_t in regression [2] is defined by

$$v_t = \frac{1-b}{1-b\phi}\phi\eta_{1,t}. \quad [7]$$

Then, the covariance matrix Σ of the innovations u_t and v_t in the bivariate regressions [1]-[2] has elements given by

$$\sigma_{uv}(\phi, b) = \left(\frac{1-b}{1-b\phi}\right)^2 \frac{\phi}{1-b}\sigma_1^2,$$
$$\sigma_u^2(\phi, b) = \left(\frac{1}{1-b\phi}\right)^2 (\sigma_1^2 + (1-b\phi)^2\sigma_2^2), \quad [8]$$
$$\sigma_v^2(\phi, b) = \left(\frac{1-b}{1-b\phi}\right)^2 \phi^2\sigma_1^2,$$

which are functions of the two economic parameters, the persistence parameter in the fundamental process and the discount factor[11]. Note that v_t is positively correlated with u_t, reflecting an endogenous feedback from forecasting errors to the future values of the regressor.

3. Information content in the spot return regression

We now study the influence of the magnitude of two key quantities, $\frac{\sigma_{uv}}{\sigma_v^2}$ and $\frac{\sigma_u}{\sigma_v}$, on the information content of the spot regression. We show that the scaled correlation $\frac{\sigma_{uv}}{\sigma_v^2}$ affects the magnitude of the finite sample bias and the noise-to-signal ratio $\frac{\sigma_u}{\sigma_v}$ mainly determines the variance of the estimated slope coefficient, $\hat{\beta}$. Further, the magnitude of the relative quantity, $\frac{\sigma_{uv}}{\sigma_v^2} / \frac{\sigma_u}{\sigma_v}$, which is the contemporaneous correlation, reflects the endogeneity problem and affects the over-rejections of the *t*-test along with the persistent regressor.

(10) Without loss of generality, we assume that the constant terms are equal to zero in the random walk and integrated AR(1) fundamental processes.

(11) See also Moon and Velasco (2014) who derive similar expressions from the present value model of stock prices.

Table 2: Estimates of contemporaneous correlations ($\hat{\rho}$) and regressor persistence ($\hat{\varphi}$) in foreign exchange markets

Series	$\frac{var\ (y_t)}{var\ (x_t)}$	$\hat{\varphi}$	s.e.	$\hat{\rho}$	T
Panel A. The entire sample period					
ATS	145.33	0.88	0.05	-0.15	275
BEF	152.02	0.73	0.07	-0.19	275
CAD	160.91	0.87	0.04	0.07	420
DKK	90.62	0.78	0.07	-0.13	420
FRF	95.89	0.70	0.10	-0.22	275
DEM	144.89	0.94	0.03	-0.08	275
IEP	84.54	0.25	0.22	-0.08	236
ITL	49.41	0.81	0.05	0.01	275
JPY	218.04	0.93	0.03	-0.11	391
NLG	142.66	0.87	0.05	-0.14	275
NOK	96.47	0.76	0.08	-0.12	420
PTE	11.49	0.83	0.11	-0.16	275
ESP	28.11	0.69	0.10	-0.25	275
SEK	80.28	0.69	0.15	-0.14	420
CHF	158.85	0.95	0.03	-0.07	420
GBP	177.43	0.90	0.04	-0.02	420
AUD	194.02	0.89	0.06	-0.04	313
NZD	79.49	0.86	0.09	0.03	313
Panel B. The subsample period					
ATS	167.54	0.98	0.02	-0.15	131
BEF	150.20	0.81	0.11	-0.17	131
CAD	218.26	0.93	0.03	0.19	276
DKK	138.79	0.77	0.11	-0.05	276
FRF	126.01	0.90	0.10	-0.29	131
DEM	143.91	0.99	0.01	0.01	131
IEP	57.68	0.05	0.30	0.00	131
ITL	155.15	0.96	0.06	-0.06	131
JPY	278.92	0.97	0.02	-0.06	276
NLG	153.36	0.99	0.01	-0.04	131
NOK	153.14	0.69	0.15	-0.03	276
PTE	57.59	0.90	0.06	-0.01	131
ESP	99.85	0.77	0.13	-0.28	131
SEK	106.18	0.72	0.24	-0.07	276
CHF	270.42	0.96	0.02	-0.09	276
GBP	266.81	0.95	0.03	-0.01	276
AUD	276.56	0.94	0.04	0.03	276
NZD	383.75	0.85	0.05	-0.12	276

Note: ρ and φ are estimated from the bivariate regressions [1]-[2] using monthly foreign excess returns and forward premiums. See also Note in Table 1.

Consider the bivariate system of regressions [1]-[2] where f_t and s_t are generated from the present value model specified in the previous section. Following Stambaugh (1999), the expected sampling error of the estimated slope coefficient under normality is calculated by

$$E[\hat{\beta} - \beta_0] = \frac{\sigma_{uv}}{\sigma_v^2} E[\hat{\phi} - \phi] \approx -\frac{1}{\phi(1-b)} \frac{(1+3\phi)}{T}, \quad [9]$$

where the first order approximation to $E[\hat{\phi} - \phi]$ is defined by $-(1 + 3\phi)/T$ using the analysis of Kendall (1954, Eq. [20]). Equation [9] shows that the discount factor b critically determines the feedback scale factor, $\frac{\sigma_{uv}}{\sigma_v^2}$, and thus strongly affects the magnitude of the bias: the value of the scale factor, $1/(\phi(1-b))$, significantly increases as b goes to unity. For example, for $\phi = 0.3$, $E[\hat{\beta} - \beta_0]$, is -0.11 for $b = 0.9$, -0.22 for $b = 0.95$, and -1.08 for $b = 0.99$ for a currently available monthly sample of $T = 400$ in foreign exchange data. Monte Carlo simulations in the next section which incorporate more realistic shock processes and different data generating processes further confirm our analysis. The factor $\frac{\sigma_u}{\sigma_v} = \frac{1}{\phi(1-b)}\sqrt{1 + (1 - b\phi)^2\sigma_2^2/\sigma_1^2}$ also becomes larger as b is close to one. And we know that the variance of the regressor is close to zero for b sufficiently close to one from equation [6][12]. As a result, larger values of $\frac{\sigma_u}{\sigma_v}$ implied by the near unity value of b make the distribution of $\hat{\beta}$ wider and thus the estimated slope coefficient becomes less informative. These two results demonstrate that the present value model with near unity discount factor can produce insignificant values of the estimates very far away from the value under the null hypothesis, including the negative values observed in the data. Further, equations [6] and [8] show that the present model can generate a large magnitude of relative volatility between the spot return and the forward premium as b is sufficiently close to unity, which is consistent with the data (see the second column in Table 2).

We now discuss how this large magnitude of the bias is related to the overrejection of the conventional t-test. For this, we relate the two quantities $\frac{\sigma_{uv}}{\sigma_v^2}$ and $\frac{\sigma_u}{\sigma_v}$ to the magnitude of the contemporaneous correlation which is known to affect statistical inference. The contemporaneous correlation between u_t and v_t is the ratio between $\frac{\sigma_{uv}}{\sigma_v^2}$ and $\frac{\sigma_u}{\sigma_v}$, as defined in the following decomposition,

$$\rho = \frac{\sigma_{uv}/\sigma_v^2}{\sigma_u/\sigma_v} = \frac{1}{\sqrt{1+(1-b\phi)^2\sigma_2^2/\sigma_1^2}}. \quad [10]$$

Equation [10] shows that a large magnitude of bias by itself (summarized in $\frac{\sigma_{uv}}{\sigma_v^2}$) does not necessarily lead to severe over-rejections of the t-test relative to its nominal

(12) Although we restrict our attention for the case of constant b, its relaxation strengthens our results. For example, West (2012) assumes $b = 1 - \frac{d}{\sqrt{T}}$ where $d > 0$ is constant and shows that the conventional t-test is not consistent.

size. Rather, what matters is the ratio between $\frac{\sigma_{uv}}{\sigma_v^2}$ and $\frac{\sigma_u}{\sigma_v}$, which determines the standardized bias of the *t*-test. Equation [10] further shows that the contemporaneous correlation depends on several economic parameters. Note that in this simple setup, the correlation can be close to one if both the discount factor b and the persistence parameter ϕ are close to one. In the next section, however, we show that the correlation can be close to zero in the general setup, while preserving near unity values of both b and ϕ.

4. Monte Carlo experiments

In this section we conduct Monte Carlo simulations to examine how severely the discount factor affects the information content of the slope coefficient in the spot return regression.

We use as data generating process equation [4] for the spot exchange rate and the modified version of equation [5] for the fundamentals. To be compatible with the evidence on the persistence of the forward premium in the data, we modify the fundamental process in [5] by

$$\begin{aligned} \Delta w_t &= \Delta w_{1,t} + \eta_{2,t}, \\ \Delta w_{1,t} &= \phi \Delta w_{1,t-1} + \eta_{1,t} + \theta \eta_{1,t-1}, \end{aligned} \qquad [11]$$

where $\theta < 0$ and $0 < \phi + \theta < 1$[13]. For robustness, we also consider an alternative fundamental process given by

$$\begin{aligned} w_t &= w_{1,t} + w_{2,t}, \\ w_{1,t} &= \phi w_{1,t-1} + \eta_{1,t}, \\ w_{2,t} &= w_{2,t-1} + \eta_{2,t}, \end{aligned} \qquad [12]$$

where $0 < \phi < 1$ and both $\eta_{1,t}$ and $\eta_{2,t}$ are *iid* zero mean normal distributions with variance σ_1^2 and σ_2^2, respectively.

For both specifications, we consider nine combinations of the following parameter values ϕ = [0.9, 0.95, 0.99] and b = [0.9, 0.95, 0.97]. For the first specification, we set (ϕ, θ) = [(0.9, –0.8), (0.95, –0.85), (0.99, –0.9)]. With this parameterization and by varying the magnitude of the relative variance σ_2/σ_1, we obtain that the absolute value of ρ is around [0.2, 0.25], which covers its upper bound in the data.

4.1. Results

Table 3 presents simulation results using 10,000 repetitions. Panel A reports the results from the case in which s_t is generated using equations [4] and [11] and Panel B presents the results from the case in which s_t is generated using [4] and [12]. The conventional *t*-test is conducted for 1, 5, 10% significant levels against left-tail and right-

(13) We omitted our analysis with this generalization in Section 3 for the sake of simplicity but it is available upon request.

Table 3: Simulation results

Parameter values			t-test size		Percentiles of the distribution of $\hat{\beta}$						
b	ϕ	θ	5L	5R	10%	20%	50%	80%	90%	$\hat{\rho}$	$\hat{\varphi}$
Panel A. First specification of fundamentals											
0.90	0.90	-0.80	0.06	0.04	-2.51	-1.41	0.79	2.81	3.87	0.26	0.89
0.90	0.95	-0.85	0.07	0.04	-1.42	-0.63	0.83	2.17	2.84	0.24	0.94
0.90	0.99	-0.90	0.08	0.03	-0.49	0.00	0.82	1.55	1.94	0.23	0.98
0.95	0.90	-0.80	0.06	0.04	-5.70	-3.57	0.63	4.55	6.57	0.23	0.89
0.95	0.95	-0.85	0.07	0.04	-3.22	-1.82	0.72	3.06	4.25	0.23	0.94
0.95	0.99	-0.90	0.07	0.03	-1.53	-0.68	0.72	2.00	2.68	0.20	0.98
0.97	0.90	-0.80	0.07	0.04	-6.51	-4.09	0.45	4.63	6.72	0.33	0.89
0.97	0.95	-0.85	0.07	0.04	-5.05	-3.08	0.58	3.92	5.60	0.24	0.94
0.97	0.99	-0.90	0.08	0.03	-2.17	-1.13	0.59	2.14	2.94	0.24	0.98
Panel B. Second specification of fundamentals											
0.90	0.90	0.00	0.04	0.06	-0.57	-0.01	1.07	2.22	2.82	-0.17	0.89
0.90	0.95	0.00	0.04	0.06	-1.03	-0.30	1.13	2.68	3.53	-0.19	0.94
0.90	0.99	0.00	0.03	0.08	-3.50	-1.65	1.86	5.78	8.13	-0.22	0.98
0.95	0.90	0.00	0.04	0.06	-0.60	-0.03	1.07	2.24	2.85	-0.17	0.89
0.95	0.95	0.00	0.04	0.06	-1.38	-0.52	1.13	2.90	3.85	-0.17	0.94
0.95	0.99	0.00	0.03	0.07	-4.00	-1.97	1.88	6.10	8.63	-0.21	0.98
0.97	0.90	0.00	0.05	0.06	-0.76	-0.13	1.07	2.35	3.01	-0.16	0.89
0.97	0.95	0.00	0.04	0.06	-1.10	-0.34	1.14	2.73	3.60	-0.19	0.94
0.97	0.99	0.00	0.04	0.07	-4.68	-2.40	1.88	6.49	9.30	-0.19	0.98

Note: To generate the simulated data, the first specification uses equations [4] and [11] and the second specification uses [4] and [12], respectively. The t-test is conducted for the 5% significant level against either left-tail (5L) or right-tail (5R) alternatives. 10%, ..., 90% denote percentiles of the distribution of $\hat{\beta}$: for example, 10% means the 10th percentile. $\hat{\rho}$ is the (averaged) estimate of the contemporaneous correlation between u_t and v_t in the bivariate regressions [1]-[2] and $\hat{\varphi}$ is the estimated slope coefficient in regression [2].

tail alternatives, respectively. To conserve space, we only report the results at the 5% significant level for left- and right-tail alternatives. We also report the five different percentiles of the distribution of $\hat{\beta}$, the estimates of the contemporaneous correlation, $\hat{\rho}$, and the estimates of the first order persistence parameter of the regressor, $\hat{\varphi}$.

Overall, we find that the distribution of $\hat{\beta}$ is very disperse in that we observe negative values of the estimates up to 40th percentiles under the null hypothesis of $\beta = 1$. Nevertheless, the size of the *t*-test based on the conventional asymptotic critical values is close to its nominal value.

In all the specifications considered in this paper, size distortions do not appear despite of strong persistence in the regressor. This is mainly because we designed the simulations so that the contemporaneous correlation between the disturbance to the dependent variable and to the regressor is very low, following the evidence in the data. The absolute values of the estimate of ρ in our simulations are close to the maximum value obtained from the data (see the sixth column in Table 2), implying that the overrejection of the conventional *t*-test is not likely to occur in the foreign exchange rate data even if the forward premium is very persistent (see the third column in Table 2).

Nevertheless, the magnitude of the bias for the estimation turns out to be large. For example, in the first model with ($b = 0.9$, $\phi = 0.9$, and $\theta = -0.8$), the estimated slope coefficient, $\hat{\beta}$, is 0.79 at the median, –1.41 at the 20th percentile, and 2.81 at the 80th percentile, while the value is one under the null hypothesis, confirming that the distribution of the estimated slope coefficient is very disperse [See the first row of Panel A in Table 3]. The width of the distribution becomes larger as the discount factor b is close to unity: for example, the median estimated slope coefficient is 0.63 and 0.45 for $b = 0.95$ and 0.97, respectively, holding other things constant. This result is robust with respect to different parameter values of ϕ and θ. We also find similar results from the second model with equations [4] and [12]. We also increase sample size to see how fast this small sample bias disappears. As reported in Table 4, although the distribution of $\hat{\beta}$ is narrower as sample size increases, the above conclusion still holds true even for $T = 1000$.

5. Conclusions

This paper shows that the estimates of the spot return regression may not convey useful information about the predictability of excess returns if exchange rates are generated from the typical present value model where the discount factor is near unity. The main reason is that the model induces imprecise estimation of the slope coefficient in the regression accompanied by a large magnitude of the variance of the estimate. Despite of large magnitude of the bias in the estimation, the size of the conventional *t*-test based on the asymptotic critical values is close to the nominal value because the contemporaneous correlation is very small in the foreign exchange rate data. Empirical evidence on the near unity value of the discount factor and on the large magnitude of the relative variance between the spot return and the forward premium further supports our analysis.

Table 4: Simulation results: T = 1,000

Parameter values			Size		Percentiles of the distribution of $\hat{\beta}$						
b	ϕ	θ	5L	5R	10%	20%	50%	80%	90%	$\hat{\rho}$	$\hat{\varphi}$
Panel A. First specification of fundamentals											
0.90	0.90	-0.80	0.06	0.05	-1.06	-0.38	0.91	2.19	2.80	0.26	0.90
0.90	0.95	-0.85	0.06	0.04	-0.36	0.07	0.93	1.75	2.15	0.24	0.95
0.90	0.99	-0.90	0.07	0.04	0.27	0.51	0.93	1.31	1.52	0.23	0.99
0.95	0.90	-0.80	0.05	0.05	-2.93	-1.64	0.84	3.30	4.50	0.23	0.90
0.95	0.95	-0.85	0.06	0.04	-1.39	-0.62	0.87	2.32	3.02	0.23	0.95
0.95	0.99	-0.90	0.06	0.04	-0.23	0.17	0.88	1.56	1.92	0.20	0.99
0.97	0.90	-0.80	0.06	0.05	-3.34	-1.91	0.76	3.40	4.67	0.33	0.90
0.97	0.95	-0.85	0.06	0.04	-2.41	-1.32	0.81	2.87	3.88	0.24	0.95
0.97	0.99	-0.90	0.07	0.04	-0.54	-0.04	0.84	1.66	2.08	0.24	0.99
Panel B. Second specification of fundamentals											
0.90	0.90	0.00	0.05	0.05	0.01	0.36	1.03	1.72	2.08	-0.17	0.90
0.90	0.95	0.00	0.05	0.06	-0.24	0.18	1.07	1.97	2.43	-0.19	0.95
0.90	0.99	0.00	0.04	0.06	-1.48	-0.51	1.34	3.34	4.48	-0.22	0.99
0.95	0.90	0.00	0.05	0.05	-0.01	0.35	1.03	1.73	2.10	-0.17	0.90
0.95	0.95	0.00	0.05	0.05	-0.44	0.05	1.07	2.11	2.62	-0.17	0.95
0.95	0.99	0.00	0.04	0.06	-1.74	-0.68	1.35	3.51	4.72	-0.21	0.99
0.97	0.90	0.00	0.05	0.05	-0.11	0.28	1.03	1.79	2.19	-0.16	0.90
0.97	0.95	0.00	0.05	0.05	-0.28	0.16	1.07	2.00	2.47	-0.19	0.95
0.97	0.99	0.00	0.04	0.06	-2.08	-0.89	1.36	3.74	5.05	-0.19	0.99

Note: To generate the simulated data, the first specification uses equations [4] and [11] and the second specification uses [4] and [12], respectively. The *t*-test is conducted for the 5% significant level against either left-tail (5L) or right-tail (5R) alternatives. 10%, ..., 90% denote percentiles of the distribution of $\hat{\beta}$: for example, 10% means the 10th percentile. $\hat{\rho}$ is the (averaged) estimate of the contemporaneous correlation between u_t and v_t in the bivariate regressions [1]-[2] and $\hat{\varphi}$ is the estimated slope coefficient in regression [2].

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RESUMEN

En este trabajo se muestra la posibilidad de que las estimaciones obtenidas a partir de regresiones sobre las rentabilidades del mercado de divisas contengan un elevado nivel de ruido que dificulte la extracción de información útil sobre la previsibilidad del exceso de rentabilidad, en particular si los tipos de cambio se generan a partir de un modelo típico de valor presente con un factor de descuento cercano a la unidad. La razón principal es que el modelo de valor presente induce un gran sesgo en la estimación de las regresiones acompañado a su vez por una alta variabilidad de las estimaciones. También se confirma que la volatilidad y la persistencia tanto de la rentabilidad al contado como de la prima a plazo generada a partir del modelo de valor presente son consistentes con los datos.

Palabras clave: modelo de valor presente, factor de descuento, correlación contemporánea, rompecabezas de la prima a plazo.

Clasificación JEL: F31, C13.